Chapter

6 Polarization of Light

♦ 6.1. UNPOLARIZED LIGHT OR ORDINARY LIGHT

According to electromagnetic theory, light waves are electromagnetic waves. *An electromagnetic wave consists of time varying electric and magnetic fields, each field being perpendicular to the other and both are perpendicular to the direction of propagation of the wave* (Fig. 6.1).

The electric field is represented by an electric vector $\vec{E}$ and magnetic field is represented by the magnetic vector $\vec{B}$. It has been observed experimentally that all observed effects of light are produced by electric vector $\vec{E}$ only, hence light is represented by the electric vector $\vec{E}$ (The amplitude of electric field vector is very large as compared to the amplitude of magnetic field vector ($E_0 = cB_0$). Moreover, our eye is more sensitive to electric field vector than to magnetic field vector).

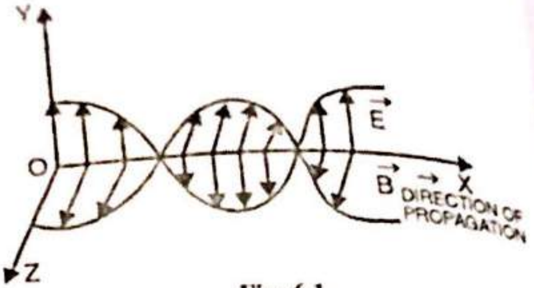


Fig. 6.1.

Light is emitted, when an electron jumps from the higher energy level to the lower energy level of an excited atom. A source of light contains millions of excited atoms. Each excited atom emits light after 10^{-8} seconds in all possible directions. So, an ordinary light is *unpolarised i.e.*, the electric vector $\vec{E}$ keeps on changing its direction in a random manner.

♦ 6.2. REPRESENTATION OF UNPOLARIZED AND POLARIZED LIGHT

An ordinary beam of light contains waves vibrating in all possible planes with equal probability. If the direction of propagation of the beam of light is perpendicular to the plane of paper while its vibrations are in the plane, then it is represented as shown in Fig. 6.2 (*a*).

As these vibrations can be resolved into any two planes mutually perpendicular to each other, so a ray of ordinary light may be regarded as consisting of two sets of waves, one set vibrating in one direction and the other at right angles to it (Fig. 6.2 (*b*)). Ordinary or unpolarized light is represented as shown in (Fig. 6.2 (*c*)). Arrows represent the set of waves vibrating in the plane of paper while dots represent the set of waves vibrating perpendicular to the plane of paper.

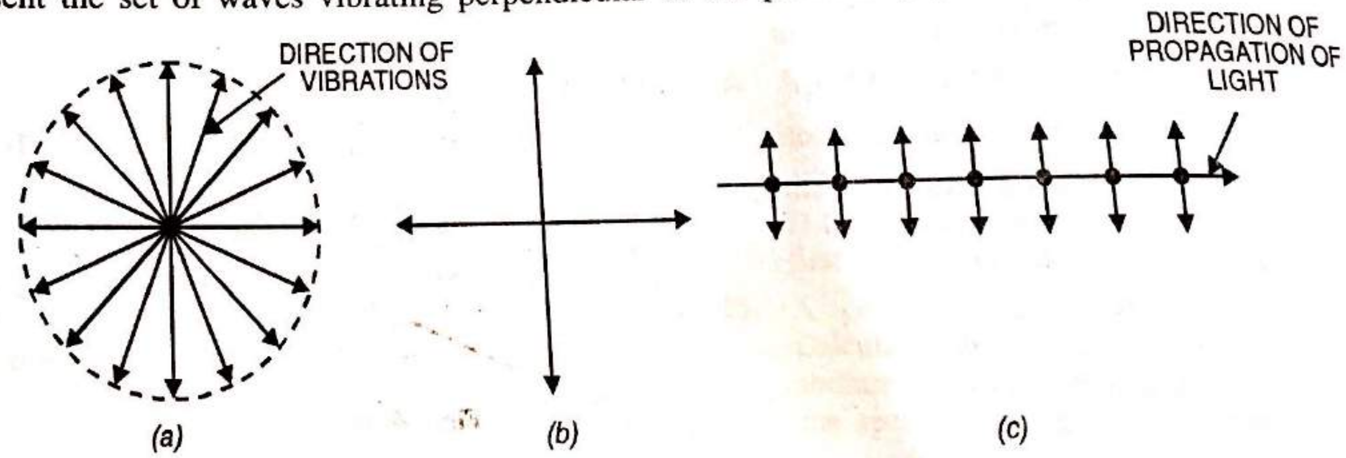


Fig. 6.2.

6.3. POLARIZATION OF LIGHT

The phenomenon of restricting the vibration of light (i.e. electric field vector $\vec{E}$) in a particular direction in a plane perpendicular to the direction of propagation of light is known as polarization of light.

The plane in which the vibration of electric field vector $\vec{E}$ takes place is known as *plane of vibration*. However, *the plane perpendicular to the plane of vibration is known as plane of polarization* (Fig. 6.3).

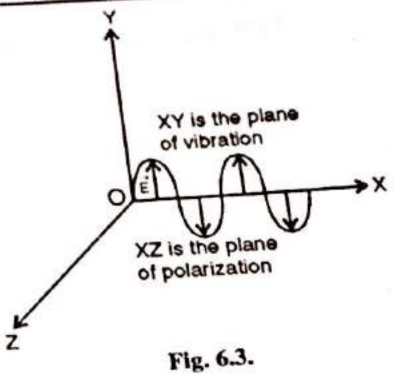


Fig. 6.3.

6.4. PLANE CIRCULARLY AND ELLIPTICAL POLARIZATION

Plane polarized Light : When the electric field vector $\vec{E}$ oscillates only in one direction in a plane, then the light is known as linearly polarized light or *plane polarised light*. In plane polarised light, the vibrations of electric vector $\vec{E}$ are along one direction perpendicular to the direction of the propagation of light. If the vibrations of $\vec{E}$ are in the plane of paper, they are represented by arrows as shown in figure 6.4(*a*). If the vibrations of $\vec{E}$ are perpendicular to the plane of paper, they are represented by dots as shown in figure 6.4(*b*). Thus, a plane polarised light is represented as shown in figure 6.4.

Fig. 6.4. (Plane polarised light)

It may be noted that the intensity of polarized light is half the intensity of unpolarized light.

Elliptically polarized Light

If in a polarized light, the electric vector $\vec{E}$ describes an ellipse, then the polarized light is known as *elliptically polarized light*. In this case, the direction of oscillation of the electric vector rotates periodically but the magnitude of electric vector changes within certain maximum and minimum limits. Elliptically polarized light is represented as shown in figure 6.5.

Circularly polarized light

If in a polarized light, the electric vector $\vec{E}$ describes a circular path, then the polarized light is known as *circularly polarized light*. In this case, the direction of oscillation of the electric vector rotates periodically but the magnitude of electric vector remains the same. Circularly polarized light is represented as shown in figure 6.6.

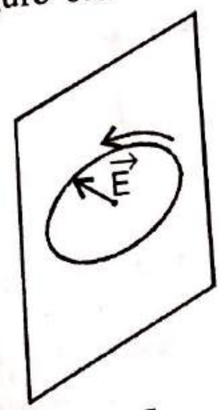


Fig. 6.5.

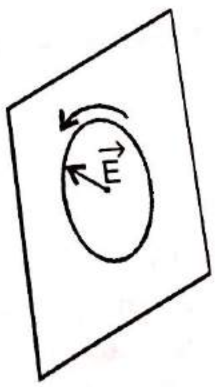


Fig. 6.6.

♦ 6.5. POLARISATION BY REFLECTION

Polarisation by reflection is the simplest method to produce plane polarised light. This method was discovered by E.L. Malus in 1806. According to Malus when a beam of ordinary light is reflected from a transparent medium (like glass), the reflected light is partially polarised. The degree of polarisation depends on the incident angle. The reflected light is almost plane polarised at a certain angle of incidence (depending on the nature of the reflecting surface), called **angle of polarisation** or **polarising angle** of the medium. The angle of polarisation for ordinary glass is 57·5°. *The vibrations of the plane polarised reflected light are perpendicular to the plane of incidence and hence the reflected light is said to be plane-polarised in the plane of incidence.*

Experiment

Consider a beam of ordinary light incident along PQ on a reflecting surface AB of the transparent medium. Let the light is reflected along QR. Place a tourmaline crystal C (Fig. 6.7) in the path of the reflected light and rotate this crystal about QR as axis. The variation in the intensity of transmitted light indicates that the reflected light is partially polarized. Now fix the tourmaline crystal in the position of maximum intensity. Change the angle of incidence of the incident beam of light by changing the inclination of the reflecting surface AB till the intensity of the reflected light after transmission through the crystal is zero. The angle of incidence at which this occurs (*i.e.*, the plane polarised light in the reflected beam is maximum) is called **angle of polarisation.** If the tourmaline crystal is rotated about QR, the intensity of the reflected beam after transmission through the crystal varies between *maximum* and *minimum* twice in each rotation. It is maximum or minimum depending on whether the crystallographic axes of the crystal is perpendicular to or lies in the plane of incidence. This shows that the light vibrations in the plane polarised reflected beam are perpendicular to the plane of incidence. This is the simple method for production and detection of the plane polarised light.

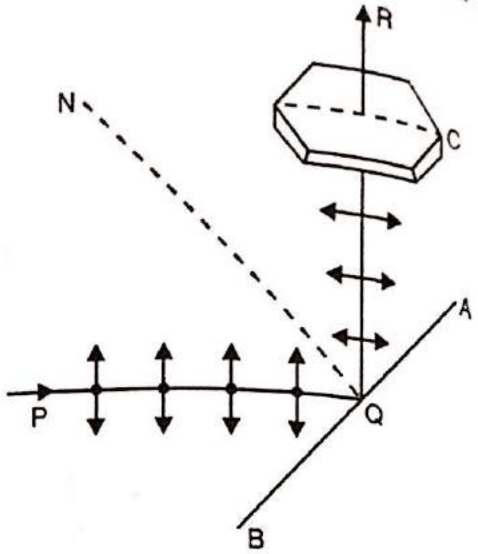


Fig. 6.7.

Explanation. The production of polarised light by reflection from any transparent medium (glass, say) can be explained as follows.

Any vibration of an unpolarized light beam can be resolved into two components, one along the horizontal plane and the second along the vertical plane. Thus, an unpolarized light may be resolved into two plane-polarized light beams at right angles to each other as shown in the Fig. 6.8. Here, the horizontal component is E_x=E cos θ, which is shown by dot and the vertical component is E_y= E sin θ, which is shown by an arrow.

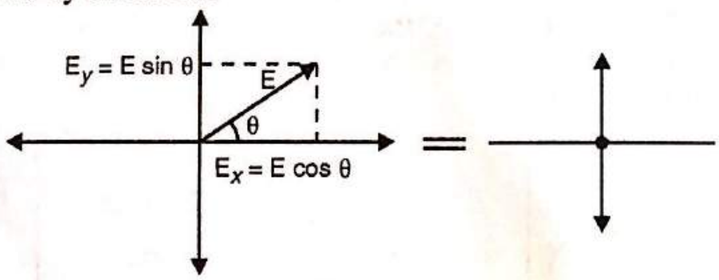


Fig. 6.8.

Let us consider an incident beam AB falling on the surface XY of the glass slab. BC is the reflected beam and BD is the refracted beam (Fig. 6.9 (*a*)). The incident beam can be resolved into two components, one parallel to the plane of incidence and the other perpendicular to the plane. The light due to the perpendicular component is mainly reflected but the remainder light mainly because of the parallel component is refracted into the glass. Thus, the light which is reflected by the glass is partially plane polarized.

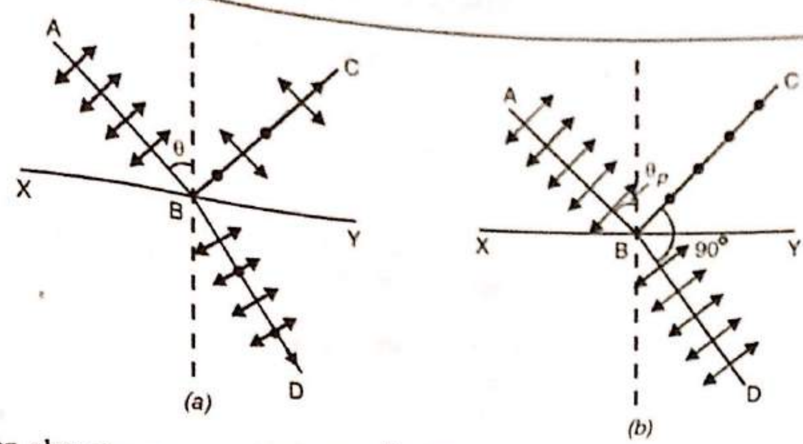


Fig. 6.9.

As we go on changing the angle of incidence, a stage comes when the angle between the reflected and refracted beam is 90° as shown in Fig. 6.9 (*b*). Thus, the reflected beam consists only of one component. This reflected beam although of low intensity is plane polarized with its plane of vibrations at right angles to the plane of incidence. *The angle of incidence for which the reflected beam is completely plane polarized is known as the* **polarizing angle** (θ_p)). The refracted beam is partially plane polarized. For glass (n = 1·52) this angle is 57° which varies slightly with the wavelength of the incident light, since the refractive index depends upon the wavelength. Thus complete polarization is possible with only monochromatic light and not with white light. The polarization of the reflected beam can easily be verified by an analyser.

♦ 6.6. BREWSTER'S LAW

Sir Brewster in 1811 discovered that there is a simple relation between angle of polarisation and the refractive index of the medium. According to Brewster *the refractive index of the refractive medium is numerically equal to the tangent of the angle of polarisation.*

i.e.,
$$n = \tan \theta_p \quad \text{...(1)}$$

where θ_p is the polarising angle. This is called **Brewster'sLaw.**

A direct consequence of this law is that when light is incident at the polarizing angle, the reflected and refracted rays are mutually perpendicular to each other.

Let AB be the unpolarised light incident at the polarising angle θ_p on the boundary XY separating glass and air (Fig. 6.10). BC is the reflected ray and BD is the refracted ray. The reflected ray is completely polarised whereas the refracted ray is partially polarised.

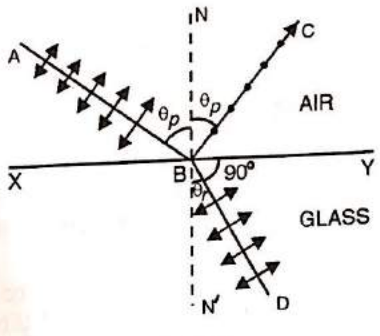


Fig. 6.10.

According to Snell's law, $n = \dfrac{\sin \theta_p}{\sin \theta_r}$ ...(1)

According to Brewster's law, $n = \tan \theta_p = \dfrac{\sin \theta_p}{\cos \theta_p}$ ...(2)

From (1) and (2), we have

$$\sin \theta_r = \cos \theta_p = \sin (90° - \theta_p)$$

or $\theta_r = 90° - \theta_p$ or $\theta_p + \theta_r = 90°$

Thus, the reflected and refracted rays are mutually perpendicular to each other if the light is incident at the polarising angle.

This is the required relation ...

◆ 6.8. MALUS LAW

This law states that *the intensity of the polarized light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polarizer.*

In Fig. 6.11, let OQ = A be the amplitude of the vibrations transmitted by the polarizer and θ be the angle between the planes of the polarizer and the analyser.

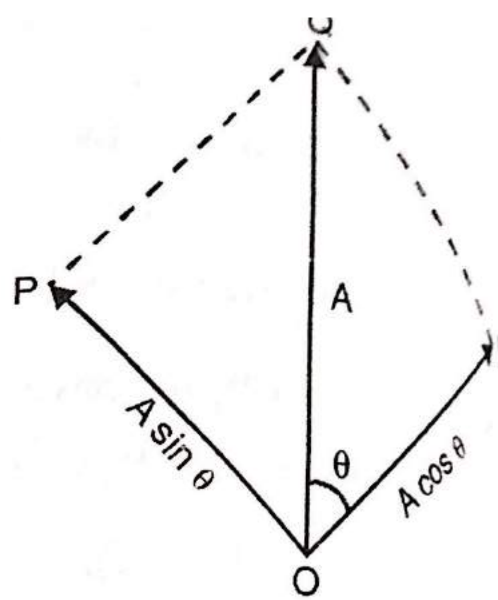


Fig. 6.11.

Resolve A into two components :

(*i*) A cos θ along OV (*i.e.*, parallel to the plane of transmission of analyser).

(*ii*) A sin θ along OP (*i.e.* perpendicular to the plane of the transmission of analyser).

Only A cos θ component is transmitted through the analyser.

∴ Intensity of the transmitted light through the analyser

$$I = (A \cos \theta)^2 = A^2 \cos^2 \theta$$

But $A^2 = I_0$ (*i.e*, intensity of incident polarised light).

∴ $$I = I_0 \cos^2 \theta \qquad ...(i)$$

which is **Malus law.**

(*i*) When $\theta = 0°$ *i.e.*, the planes of polarizer and analyser are parallel, then $\cos \theta = \cos 0° = 1$

Hence $I = I_0$.

(*ii*) Whe $\theta = \frac{\pi}{2}$, the planes of polarizer and analyser are perpendicular to each other, then

$$I = I_0 \left(\cos \frac{\pi}{2} \right)^2 = 0$$

... Through wh

We know,

$$n = \frac{c}{\upsilon}$$

Since n is constant for O-ray and hence the speed of O-ray in a crystal is same in all directions. But as 'n' varies for E-light, therefore, the speed of E-light in a crystal is different in different directions.

♦ 6.10. CALCITE CRYSTAL

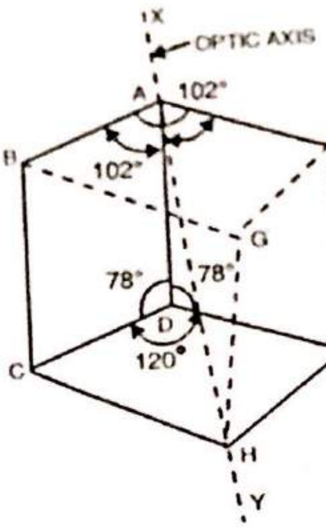


Fig. 6.13.

A calcite crystal also known as *iceland spar* is *transparent to visible as well as ultraviolet light*. Its chemical composition is calcium carbonate ($CaCO_3$) and occurs in nature in different forms, all of which break up into simple rhombohedron as shown in Fig. 6.13. Each of six faces of the crystal is a parallelogram whose angles are 78° and 102° nearly. At the two diametrically opposite corners such as A and H (Fig. 6.13), the angles of the three faces meeting there are obtuse. These corners are called **blunt corners.** At each of the remaining six corners one of the angles is obtuse whereas the other two are acute.

Optic Axis

There is a particular direction through the calcite crystal along which the speed of ordinary and extra-ordinary rays is the same (*i.e.*, along which there is no double refraction). This direction is called **optic axis.** For example, a line passing through any one of the blunt corners (A and H) and equally inclined to the three edges meeting there, represents the direction of *optic axis*. Any other line parallel to it is an identically equivalent axis in all respects.

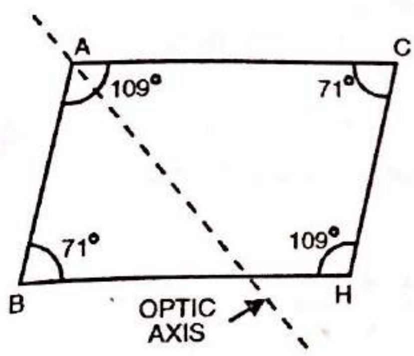


Fig. 6.14

Principal Section or principle plane

A plane containing the optic axis and perpendicular to the pair of opposite faces of the crystal is called **principal section** or **principal plane** of the crystal. ABHC is the principal section or plane for the side faces of the crystal as shown in Fig. 6.14. A principal section always cuts the surfaces of a calcite crystal in a parallelogram having angles of 71° and 109°.

♦ 6.11. TYPES OF CRYSTALS

Uni-axial Crystals Crystals are classified as either **uniaxial** or **bi-axial crystals.**

Crystal such as *quartz, calcite, tourmaline, ice and nitrate of soda* in which there is a single direction known as the optic axis along which all waves are transmitted with one uniform velocity while in any other direction, there are two velocities of transmission are called **uni-axial crystals.**

♦ Bi-axial crystals

The crystals such as borax, mica, topaz, aragonite and selenite in which there are two directions of ne uniform velocity (*i.e.*, they have two optic axes) are known as **bi-axial crystals.**

♦ 6.12. NEGATIVE AND POSITIVE CRYSTALS

Uni-axial crystals are of two types : *negative crystal* and *positive crystal.*

Negative crystal : In a negative crystal, the speed of ordinary ray (υ_o) is less than the speed of an xtra-ordinary ray (υ_e). Since, $n = \frac{c}{\upsilon}$, so $n_e < n_o$ because $\upsilon_e > \upsilon_o$. Thus, the refractive index of a egative crystal for an extra-ordinary ray is les than for ordinary ray.

The example of a negative crystal is **calcite**. In a negative crystal :

(*i*) The speed of O-ray is constant in all directions.

(*ii*) The speed of E-ray varies with direction. It is minimum and equal to the velocity of O-ray along the optic axis. It is maximum in a direction perpendicular to the optic axis.

(*iii*) The refractive index for O-ray is greater than that of E-ray (*i.e.*, $n_o > n_e$).

Positive Crystal

In a positive crystal, the speed of ordinary ray (υ_o) is greater than the speed of an extra-ordinary ray (υ_e). Since, $n = \frac{c}{\upsilon}$, so $n_e > n_o$ because $\upsilon_o > \upsilon_e$. Thus, the refractive index of a positive crystal for extra-ordinary ray is greater than for ordinary ray. The example of a positive crystal is **quartz**.

In a positive crystal :

(*i*) The speed of O-rays is constant in all directions.

(*ii*) The speed of E-ray varies with the direction. It is maximum and is equal to the velocity of O-ray along optic axis. It is minimum in a direction perpendicular to the optic axis.

(*iii*) The refractive index of E-ray is greater than that of O-ray (*i.e.*, $n_e > n_o$).

♦ 6.13. POLARIZATION BY DOUBLE REFRACTION

The ray of ordinary or unpolarized light is split into O and E rays when it is passed through doubly refracting crystal. Both O and E rays are plane polarised having vibrations perpendicular to each other. Thus, polarization of light by double refraction can be demonstrated by allowing a beam of light normally through a pair of calcite crystal and rotating the second crystal about incident ray as axis. The following phenomena are observed :

(*i*) When the principle plane of the two crystals are parallel to each other, two images E_1 and O_1 are seen (Fig. 6.15 (*a*)). These two images are separated by a distance equal to the sum of the displacement produced by each crystal when used separately.

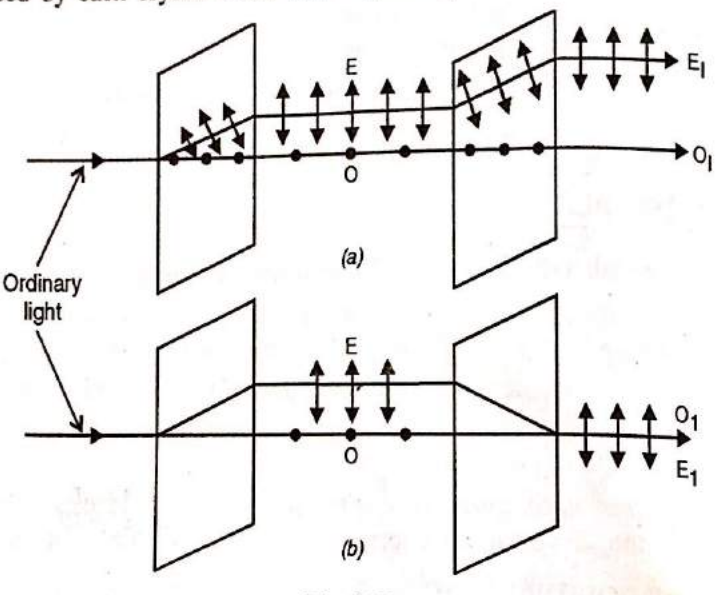


Fig. 6.15.

(*ii*) At 90° rotation, we again have two beams but the images change roles. The ordinary of the first becomes the extra-oridnary of the second and vice-versa.

(*iii*) At 180° rotation, the principle planes of the two crystals are once again parallel to each other, but having optic axes oriented in opposite directions. The image O_1 and E_1 come together to form one single image at the centre, provided the crystals are of equal thickness (Fig. 6.15 (*b*)).

7.12. Nicol Prism

It is an optical device used for producing and analysing plane polarized light. It was invented by William Nicol in 1828.

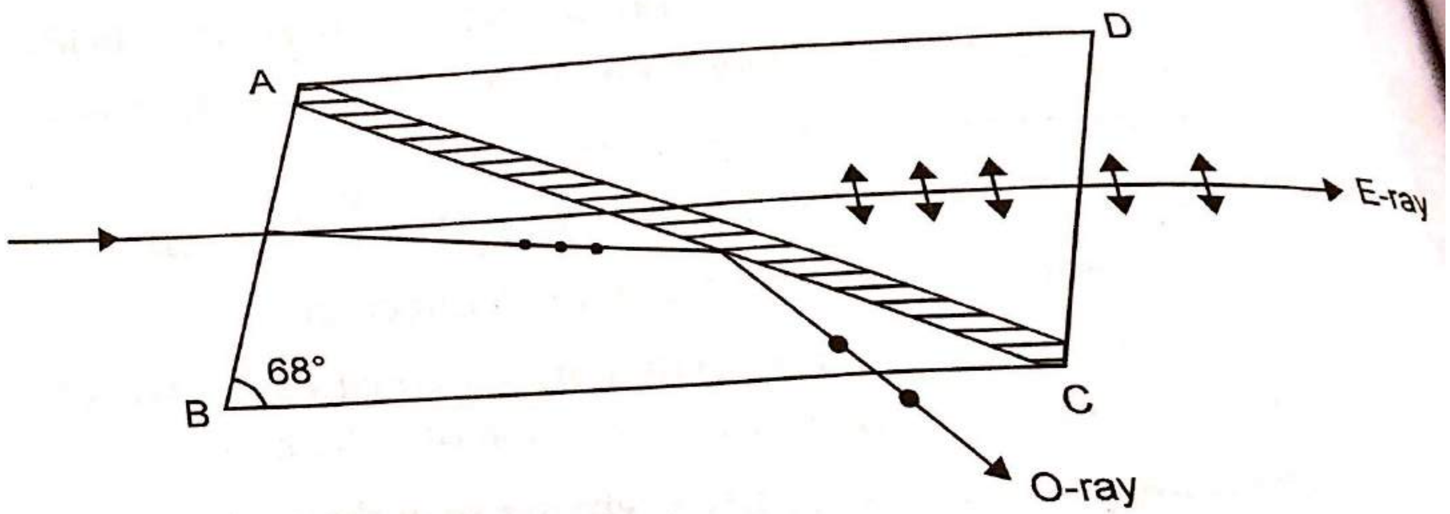


Fig. 12

When light is passed through a doubly refracting crystal, it splits up into ordinary and extraordinary rays. Both these rays are plane polarized. The nicol prism is made in such a way that it eliminates one of the two rays by total internal reflection. It is generally found that O-ray is eliminated and only E-ray is transmitted through the prism.

The nicol prism is constructed from a calcite crystal whose length is nearly three times its breadth. The end faces of the crystal are cut such that the angles in the principal section become 68° and 112° instead of 71° and 109°. The crystal is then cut along the diagonal. The two cut surfaces are grounded and polished and then cemented together with a layer of canada balsam whose refractive index lies between the refractive indicies for the O and E-rays for calcite [$\mu_0 = 1.658$, $\mu_b = 1.55$ and $\mu_e = 1.486$]

It is clear that canada balsam acts as a rarer medium for O-ray and denser medium for E-ray. The O-ray on reaching the layer of canada balsam is totally reflected because the angle of incidence on the balsam surface is greater than the critical angle which is about 69°. The E-ray is not affected and is therefore transmitted through the prism. The face where the O-ray is incident is blackened so that it is completely absorbed. The E-ray coming out of the prism is plane polarized with vibrations parallel to the principal section. Thus Nicol prism acts as a polarizer.

The Nicol prism is the most widely used polarizing device. Nicols are good polarizers but they are expensive and have a limited field of view of about 28°. They cannot be used for highly convergent or divergent beams.

The Nicol prism can be used both as a polarizer and an analyser. When two Nicols are arranged coaxially, the first Nicol N_1 which produces plane polarized light is called polarizer while the second Nicol N_2 which analyses the incoming light is called analyser.

When unpolarised light is incident on a Nicol prism N_1, the light coming out of it is plane polarized and has vibrations parallel to its principal section. Now this is made to pass through a second Nicol N_2, the principal section of which is parallel to the principal section of N_1. Since the vibrations of light incident on N_2 are also parallel to its principal section, therefore light is completely transmitted through N_2 as shown in fig. 13 (*a*). The intensity of emergent light will be maximum.

Now the Nicol N_2 is rotated so that its principal section becomes perpendicular to that of N_1 as shown in fig. 13 (*b*). The vibrations of light incident on N_2 will be perpendicular to the principal section of N_2 and hence no light is transmitted through N_2. In this position, the two Nicols are said to be crossed.

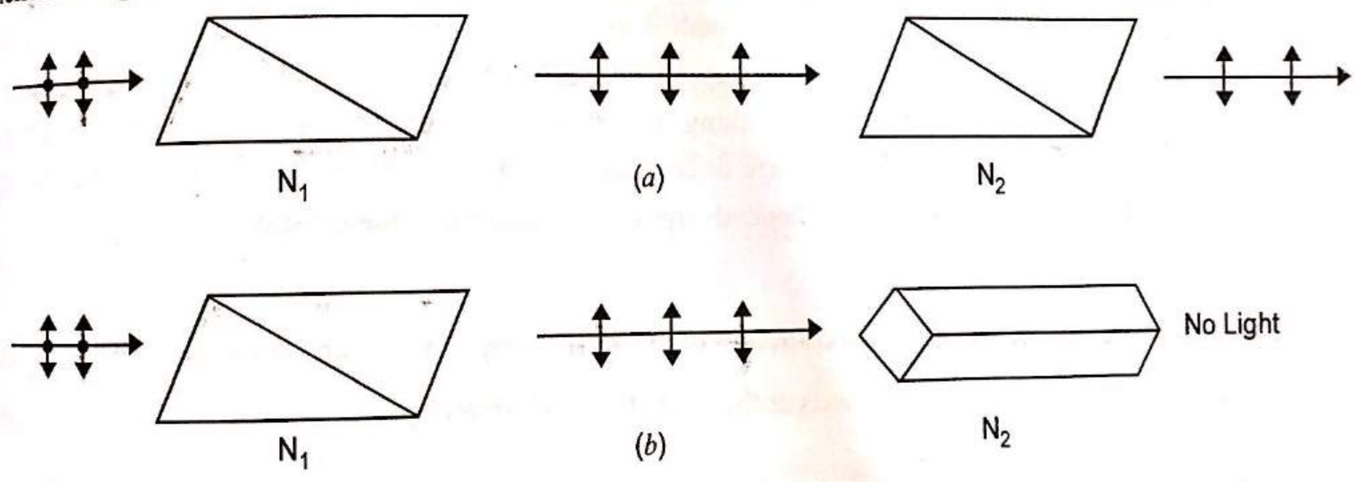


Fig. 13

When principal sections of N_1 and N_2 are neither parallel nor perpendicular but make an angle θ with each other, then according to Malus law, the intensity of transmitted light from N_2 will be proportional to $\cos^2 \theta$ *i.e.* the intensity varies between maximum and minimum values.

7.13. Circularly and Elliptically Polarized Light

In the plane polarized light, the light vector (electric field vector) vibrates along a fixed straight line perpendicular to the direction of propagation *i.e.* the orientation of light vector remains unchanged while its magnitude undergoes variations. When two plane polarised waves are superimposed, then under certain conditions, the resultant light vector may rotate. If the magnitude of light vector remains constant while its orientation varies regularly, the tip of the vector traces a circle and the light is said to be circularly polarized. If, however, both the magnitude and the orientation of the resultant light vector vary, the tip of the vector traces an ellipse and the light is said to be elliptically polarized.

Suppose a beam of plane polarized light is incident normally on a calcite crystal cut with its faces parallel to the optic axis. Let the vibrations of the incident light of amplitude A make an angle θ with the optic axis.

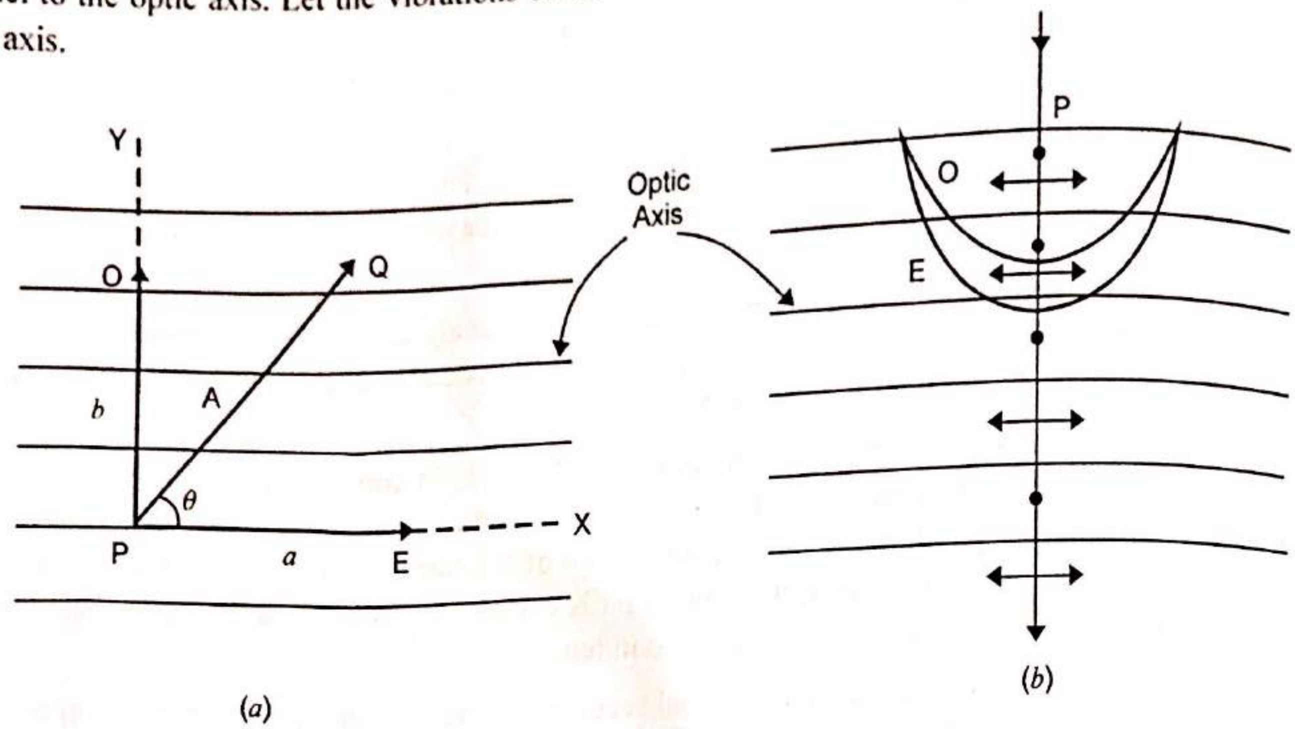


Fig. 14

On entering the crystal, the amplitude of the incident light wave will split up into two components A $\cos\theta$ along PE and A $\sin\theta$ along PO. The component A $\cos\theta$ having vibrations parallel to the optic axis forms the E-wave and the component A $\sin\theta$ having vibrations perpendicular to the optic axis forms the O-wave. The two waves travel in the crystal along the same direction but with different velocities [Fig. 14(*b*)], the E-wave being faster. Thus a phase difference δ is introduced between the two waves when they emerge out of the crystal. The value of δ depends upon the thickness of the crystal.

Put $A\cos\theta = a$ and $A\sin\theta = b$

The displacement of E-wave along x-axis at any time t is given by $x = a\sin(\omega t + \delta)$

The displacement of O-wave along y-axis at the same time t is given by $y = b\sin\omega t$

From eqn. (*ii*), $\frac{y}{b} = \sin\omega t$

$$\therefore \quad \cos\omega t = \sqrt{1-\sin^2\omega t} = \sqrt{1-\frac{y^2}{b^2}}$$

From eqn. (*i*), $$\frac{x}{a} = \sin \omega t \cos \delta + \cos \omega t \sin \delta$$

$$= \frac{y}{b}\cos\delta + \sqrt{1-\frac{y^2}{b^2}}\sin\delta$$

or $$\frac{x}{a} - \frac{y}{b}\cos\delta = \sqrt{1-\frac{y^2}{b^2}}\sin\delta$$

Squaring both sides, we have

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\delta - \frac{2xy}{ab}\cos\delta = \sin^2\delta - \frac{y^2}{b^2}\sin^2\delta$$

or $$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta = \sin^2\delta \qquad \ldots(iii)$$

This is the general equation of an ellipse. Hence the emergent light from the crystal is generally elliptically polarized.

Special Cases

(*i*) When $\delta = 0, 2\pi, 4\pi \ldots$ then $\cos\delta = 1$ and $\sin\delta = 0$

Eqn. (*iii*) becomes $$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

or $$\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0 \quad \text{or} \quad \frac{x}{a} - \frac{y}{b} = 0 \quad \text{or} \quad y = \frac{b}{a}x$$

This is equation of a straight line passing through origin and having a positive slope $\frac{b}{a}$. Thus the emergent light is plane polarized with vibrations in the same plane as in the incident light [fig. 15(*i*)].

(*ii*) When $\delta = \pi, 3\pi, 5\pi \ldots$ then $\cos\delta = -1$ and $\sin\delta = 0$

Eqn. (*iii*) gives $$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{ab} = 0$$

or $$\left(\frac{x}{a} + \frac{y}{b}\right)^2 = 0 \quad \text{or} \quad \frac{x}{a} + \frac{y}{b} = 0 \quad \text{or} \quad y = -\frac{b}{a}x$$

which is again the equation of a straight line passing through origin but with a negative slope. In this case also the emergent light will be plane polarized [Fig. 15 (*ii*)].

(*iii*) When $\delta = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots\ldots$ then $\cos\delta = 0$ and $\sin\delta = 1$

Eqn. (*iii*) reduces to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

which is the equation of an ellipse. Thus the emergent light will be elliptically polarized [Fig. 15 (*iii*)].

(*iv*) When $\delta = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots\ldots$ and $\theta = 45°$ *i.e.* the incident vibrations make an angle of 45° with the optic axis, we have $a = A \cos 45°$, $b = A \sin 45°$ $\therefore$ $a = b$

Eqn. (*iii*) becomes $x^2 + y^2 = a^2$

which is the equation of a circle of radius a. Thus the emergent light will be circularly polarized [Fig. 15 (*iv*)].

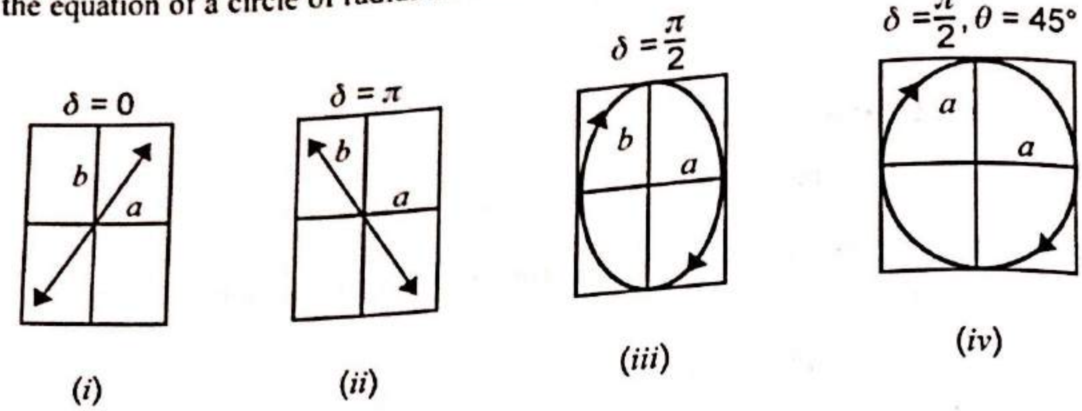


Fig. 15

7.14. Quarter Wave Plate

It is plate of doubly refracting uniaxial crystal with its refracting faces cut parallel to the optic axis. Its thickness is such that it can produce a path difference of $\frac{\lambda}{4}$ or a phase difference of $\frac{\pi}{2}$ between the ordinary and extra ordinary waves.

When a beam of monochromatic light of wavelength λ is incident normally on such a plate, it is broken into O and E waves inside the plate. Both these waves travel in the same direction but with different velocities.

In case of a negative crystal like calcite, the E-ray travels faster than O-ray so that $\mu_0 > \mu_e$, where μ_0 and μ_e are the refractive indicies of the crystal for O and E-rays respectively.

It t is the thickness of the plate, the optical paths of O and E-rays in the plate are $\mu_0\, t$ and $\mu_e\, t$ respectively. Hence the path difference between the two rays when they come out of the crystal is $\delta = (\mu_0 - \mu_e)\, t$.

For quarter wave plate, this path difference must be equal $\frac{\lambda}{4}$ *i.e.* $(\mu_0 - \mu_e)\, t = \frac{\lambda}{4}$

or $$t = \frac{\lambda}{4(\mu_0 - \mu_e)}$$

For a positive crystal like quartz, $\mu_e > \mu_0$

$$\text{or} \quad t = \frac{\lambda}{4(\mu_e - \mu_0)}$$

As the thickness depends upon the wavelength, the plate is useful only for the wavelength for which it is constructed.

A quarter wave plate is used for producing circularly and elliptically polarized light. If a plane polarized light with its vibrations making an angle of 45° with the optic axis is passed through a quarter wave plate, the emergent light is circularly polarized. If, however, the plane of vibrations of the incident plane polarized light is not inclined at an angle of 45° to the optic axis, the emergent light is elliptically polarized.

7.15. Half Wave Plate

It is a plate of doubly refracting uniaxial crystal with its refracting faces cut parallel to the optic axis. Its thickness is such that it can produce a path difference of $\frac{\lambda}{2}$ or a phase difference of π between O-ray and E-ray.

If t is the thickness of half wave plate, then for a negative crystal such as calcite ($\mu_0 > \mu_e$), the path difference between O-ray and E-ray is

$$(\mu_0 - \mu_e)\, t = \frac{\lambda}{2}$$

$$\text{or} \quad t = \frac{\lambda}{2(\mu_0 - \mu_e)}$$

where λ is the wavelength of light incident normally on the plate.

For a positive crystal such as quartz ($\mu_e > \mu_0$), the path difference is $(\mu_e - \mu_0)\, t = \frac{\lambda}{2}$

$$\text{or} \quad t = \frac{\lambda}{2(\mu_e - \mu_0)}$$

A half wave plate is used in the construction of Laurent's half shade device used in a polarimeter.

Note. To decide whether given plate is a quarter wave plate or a half wave plate, a beam of plane polarized light is allowed to fall normally on the given plate. The emergent light is then examined by a rotating Nicol. Now if the emergent light is found as elliptically or circularly polarized then the given plate is quarter wave plate but if the emergent light is found as plane polarized, the given plate is half wave plate.

7.16. Production of Plane, Circularly & Elliptically Polarized Light

1. **Plane polarized light :** A beam of monochromatic light is passed through a nicol prism. While passing through the prism, the beam is split up into ordinary and extraordinary rays. The ordinary ray is totally internally reflected back at the canada balsam layer, while the extraordinary ray passes through the nicol prism. Thus the emergent light is plane polarized.

depends on the ... ... is rotated

7.19. Specific Rotation

Liquids containing an optically active substance such as sugar solution rotate the plane of vibration of polarized light. The angle through which the plane of vibration is rotated depends upon.

(*i*) thickness of the medium (*ii*) concentration of the solution

(*iii*) wavelength of light and (*iv*) temperature

The specific rotation is defined as the rotation produced by a decimetre (10 cm) long column of a liquid containing 1 gram of the optically active substance in 1 c.c. of the solution *i.e.*

$$S_\lambda^t = \frac{10\theta}{lC}$$

where S_λ^t = specific rotation at temperature t and for wavelength λ, θ = angle of rotation, l = length of the solution in cm and C = concentration of the solution.

7.20. Laurent's Half Shade Polarimeter

A polarimeter is an instrument used for measuring the optical rotation of certain solutions. When this instrument is used to determine the quantity of sugar in a solution, it is known as saccharimeter.

Construction : The optical parts of Laurent's half shade polarimeter are shown in Fig. 18. Monochromatic light from a source S is rendered parallel by a convex lens L and made to fall upon the polarising Nicol N_1. After passing through N_1, the light becomes plane polarized. This plane polarized light passes through a half shade device H and then through a glass tube T containing the optically active solution say sugar solution. The light emerging from the solution is incident on the analysing Nicol N_2. The light is finally viewed through a telescope. The analysing Nicol N_2 can be rotated about the axis of the tube and its rotation can be measured with the help of a graduated circular scale.

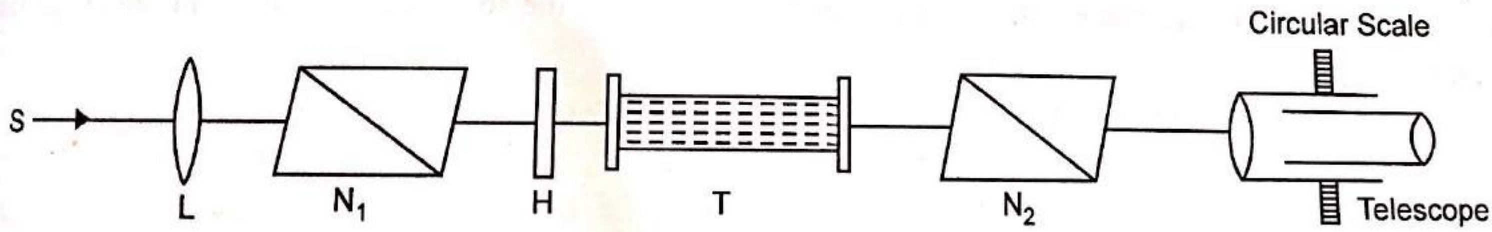


Fig. 18

Scanned with CamScanner

Working : The half shade device consists of a semicircular plate ADB of glass cemented to a semicircular plate ACB of quartz. The optic axis of the quartz plate is parallel to the line of separation AOB. The thickness of the quartz plate is such that it introduces a phase difference of π between O and E vibrations. Thus it is simply a half wave plate. The thickness of the glass plate is such that it transmits the same amount of light as transmitted by the quartz plate.

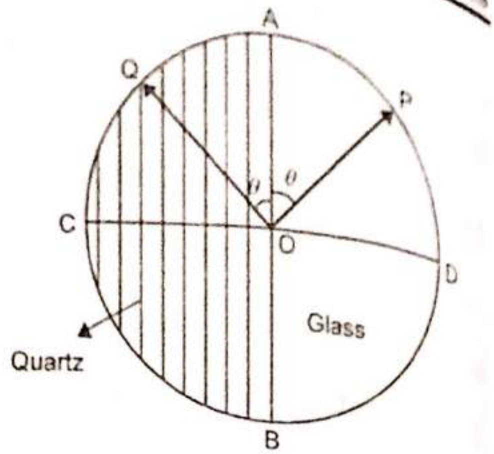


Fig. 19

Suppose the plane polarized light from N_1 is incident normally on half shade device and has vibrations along OP. On passing through the glass half, the vibrations will remain along OP but on passing through the quartz half, the vibrations will split up into E and O components. The vibrations of the O-component are along OD and those of E-component are along OA.

Since quartz plate introduces a phase difference of π between the two vibrations, the O-vibrations will occur along OC instead of OD on emerging from the plate. Thus the resultant vibration of light coming out of the quartz plate will be along OQ such that $\angle POA = \angle QOA = \theta$.

If the principal section of the analysing Nicol N_2 is held parallel to OP, the plane polarized light through glass half will pass but the light through quartz half is partially obstructed. Hence the glass half will appear brighter than the quartz half.

If the principal section of N_2 is parallel to OQ, the quartz half will appear brighter than the glass half due to the same reason.

If the principal section of N_2 is parallel to AOB, the two halves will appear equally bright. It is because the vibrations emerging out of the two halves are equally inclined to the principal section and the two components will have equal intensity.

If the principal section of N_2 is perpendicular to AOB, the two halves are said to be equally dark.

To find the optical rotation of a solution, fill the polarimeter tube with the pure solvent (water) and note the reading on the circular scale for equally dark position of half shade device.

Now fill the tube with the solution of the given substance (say sugar solution) and again note the reading on the circular scale for equally dark position of half shade device. The difference between the two readings gives the optical rotation θ. If S is the specific rotation of the solution, then concentration C of the solution is

$$C = \frac{10\theta}{l S}$$

If the concentration C of the solution is known, then specific rotation $S = \frac{10\theta}{l C}$.

...e to these

7.9. Huygen's Theory of Double Refraction

Huygen's explained the phenomenon of double refraction with the help of his principle of secondary wavelets. According to his theory :

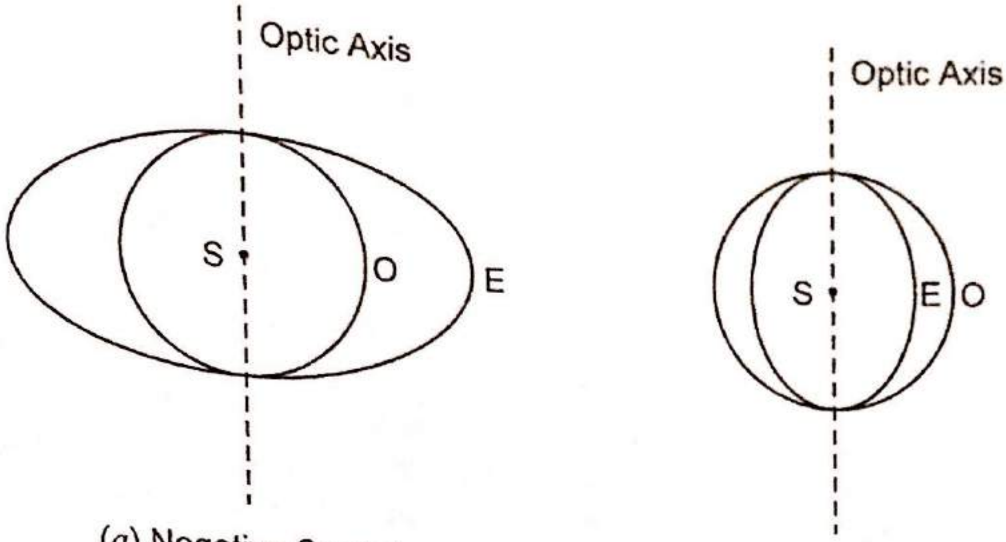
(*i*) When light waves are incident on a double refracting crystal, every point on the crystal surface becomes the origin of two secondary wavelets – ordinary and extra-ordinary, which spread out into the crystal.

(*ii*) The ordinary ray travels with the same velocity in all directions and hence its wave front is spherical.

(*iii*) The velocity of extraordinary ray varies with direction and hence its wave front is not spherical but it is ellipsoid.

(*iv*) Since the velocity of O and E-rays is the same along the optic axis, the two wave fronts sphere ellipsoid touch each other at two points. The line joining these two points is the direction of optic axis.

(*v*) For some crystals like calcite, the ordinary wave surface (sphere) lies inside the extraordinary ve surface (ellipsoid). Such crystals are called negative crystals because the refractive index for E-ray is than that for O-ray. For some other crystals like quartz, the ordinary wave surface lies outside the raordinary wave surface. Such crystals are called positive crystals because the refractive index for E-ray greater than that for O-ray.



(*a*) Negative Crystal

(*b*) Positive Crystal

Fig. 11

.10. Negative and Positive Crystals

Consider a point source of light S in a uniaxial crystal. The sphere is the wave surface for O-ray and e ellipsoid is the wave surface for E-ray as shown in Fig. 11.

For the **negative uniaxial crystals** like calcite,

(*i*) The ordinary wave surface lies inside the extra-ordinary wave surface.

(*ii*) The velocity of O-ray is constant in all directions.

(*iii*) The velocity of E-ray is minimum and equal to the velocity of O-ray along optic axis. It is maximum at right angles to the direction of optic axis.

(*iv*) The refractive index for E-ray is less than the refractive index for O-ray *i.e.* $\mu_e < \mu_0$.

(*v*) O-ray travels slower than E-ray in all directions except along the optic axis.

For the **positive uniaxial crystals** like quartz,

(*i*) The ordinary wave surface lies outside the extra-ordinary wave surface.

(*ii*) The velocity of O-ray is constant in all directions.

(*iii*) The velocity of E-ray is maximum and equal to the velocity of O-ray along optic axis. It is minimum at right angles to the direction of optic axis.

(*iv*) The refractive index for E-ray is greater than the refractive index for O-ray *i.e.* $\mu_e > \mu_0$.

(*v*) E-ray travels slower than O-ray in all directions except along the optic axis.